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Infrasonic Resonances in Natural Underground Cavities

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Several examples of acoustic resonance in the frequency range from 0.001 to 1.0 Hz have been observed in limestone caverns. In some cases, the cavern geometry is simple enough for direct application of the Helmholtz resonator theory, and good agreement is found. Three experiments are described. Spectral analysis indicates that subacoustic resonance is responsible for numerous reports of periodically fluctuating or reversing cavern winds, and that appropriate wind measurements can provide new information about chambers not accessible to exploration.

INTRODUCTION

Limestone caverns generally consist of traversable passageways typically 10–1000 m long, often interconnected, with roomlike enlargements at irregular intervals. The enlargements often have cross sectional areas orders of magnitude greater than those of the connecting passages. With these conditions, one may expect the lowest subsonic resonant frequencies to be well defined, with associated wavelengths much longer than the dimensions of the system, and the treatment outlined by Rayleigh[1,2] may be appropriate for calculating these frequencies. Typical estimates lie in the range from 0.001 to 1.0 Hz. Since surface winds are rich in excitation energy in this spectral range, subsonic oscillations should be common in natural caverns. Such oscillations are experienced as alternating winds and may be observed very sensitively by watching the motion of a candle flame or smoke. Persons familiar with caverns have, in fact, reported observations of reversing air motions of the type expected.[3–5]

The suggestion that these roughly periodic wind reversals might be acoustical in origin was made by Schmidt.[6] Many more examples of fluctuating cavern winds have been found since, some in caverns that can be treated as simple Helmholtz resonators with necks.

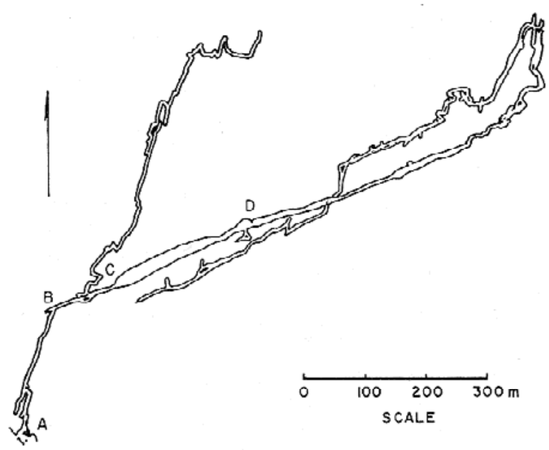

FIG. 1. Map of Cass Cave, West Virginia, adapted from Ingalls.[7] Velocity measurements were made near Point A. The passage from A to C, particularly the very small part from B to C, forms the neck of a Helmholtz resonator. The room between C and D is approximately 50 m high at C, and forms the volume of the resonator. Passages beyond Point D and the side passageway branching at C are all of much smaller volume.

Field observations made for this study were of three types: passive observation of the fundamental resonance in a cave of particularly simple geometry; recorded response to an intentionally applied transient in a cave with slightly more complicated geometry; and passive observation of a very complex cavern.

[1] J. W. Strutt Lord Rayleigh, *Theory of Sound*, (Macmillan, New York, 1878, 1896), Vol. II, pp. 309, 310; (Dover Publications, Inc., New York, 1945), pp. 187–192.
[2] J. W. Strutt Lord Rayleigh, Phil. Mag. **36**, 231–234 (1918).
[3] B. S. Faust, Nat. Speleol. Soc. Bull. **9**, 52–54 (1947).
[4] D. N. Cournoyer, Nat. Speleol. Soc. Bull. **16**, 91–93 (1954).
[5] G. W. Moore and Bro. G. Nicholas, *Speleology* (D. C. Heath and Co., Boston, 1964), p. 29.
[6] V. A. Schmidt (personal communications, 1958). See also Ref. 5, p. 29.
[7] H. Ingalls, Nat. Speleol. Soc. Bull. **21**, 21–32 (1959).

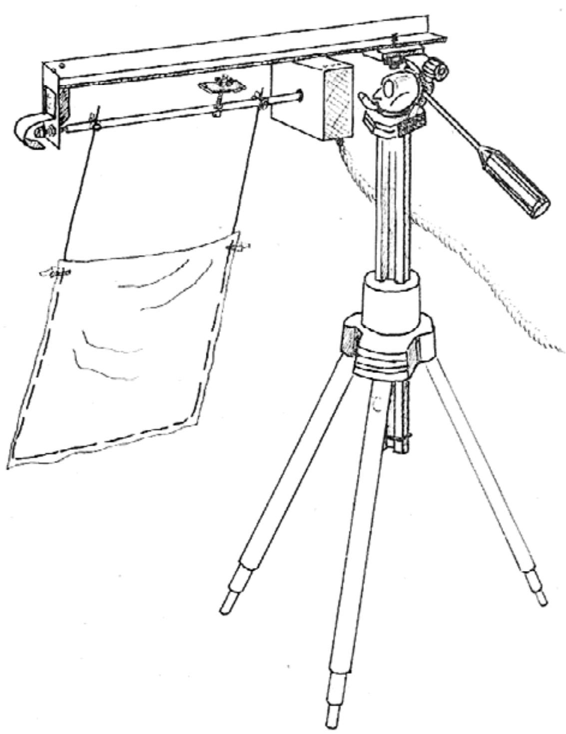

FIG. 2. Portable instrumented vane for automatic measurement of wind velocity. The small square box houses the lamp, lenses, gray scale, photodetector, and Wheatstone bridge.

I. EXPERIMENT WITH A SIMPLE RESONATOR

Cass Cave, located near Cass, West Virginia, has been described by Ingalls[7] and by Davies.[8] About 360 m from the entrance, the passage joins a room of approximately 140 000 m³ (Fig. 1). The entrance passage is not perfectly regular in cross section, but its integrated ratio of length to cross section (the reciprocal Rayleigh conductance) is about 260 m^{-1}. A Helmholtz resonator of this size and shape would have a resonant period of about 2 min, or 0.0083 Hz. Just such a resonance was observed by the author in November 1958, and again in March 1959, and in March 1969. In 1959, the air motion was made visible with cigarette smoke, and several complete reversals of the air in the entrance passageway near the large room were timed. The estimated period was close to 2 min.

Automatic recording of acoustic data in this frequency range was accomplished with a special transducer. A polyethylene bag 29 cm × 33 cm supported by a stiff frame of steel wire was rigidly attached to a pivoted brass tube to form a vane (Fig. 2). The apparatus was mounted on a photographic tripod. Each pivot consisted of a hard steel needle centered in a tapered glass sleeve and was nearly frictionless. When counterbalanced by a metal weight mounted at the top of the brass tube, the vane could be adjusted to give a large angular deflection in a wind of 10 cm/sec or less. The rotating shaft carried a transparent plastic disk that had been spray painted with a gray scale, mounted between a small lamp and a photoconductor, and the photoconductor was one arm of a Wheatstone bridge circuit. The gray scale and the bridge resistors were chosen so that the electrical imbalance signal varied linearly with wind velocity over a range of ±15° of vane motion. This imbalance signal was recorded automatically at a sampling interval of 2 sec. The apparatus was easily dismounted and carried for field use.

The instrumented vane was placed at a constricted part of the passageway about 35 m inside the entrance of Cass Cave on four occasions in July and October 1968, but there was no evidence of excitation at the 8-mHz resonance. There was a nearly constant inward air current, a "chimney" thermal current descending to some lower exit, with occasional traces of fluctuation in the 0.1-Hz range. A moderate change in barometric pressure marked the passage of a meteorological front during the October observations, but failed to excite the 8-mHz resonance.

Records obtained in March 1969 showed considerable excitation of the cave at these low frequencies. Figure 3 is a slightly smoothed copy of part of the wind-velocity record for a 13.7-min period, showing a fairly regular reversing wind. Figure 4 is a power spectrum of the data in Fig. 3, showing a marked resonance at 8.1 mHz. When a similar Fourier transform is computed for other intervals of the velocity data, the peak is consistently found at 8.1 mHz. These measurements show conclusively that a subacoustic resonance of this simple type can be identified in a limestone cavern under conditions of natural excitation.

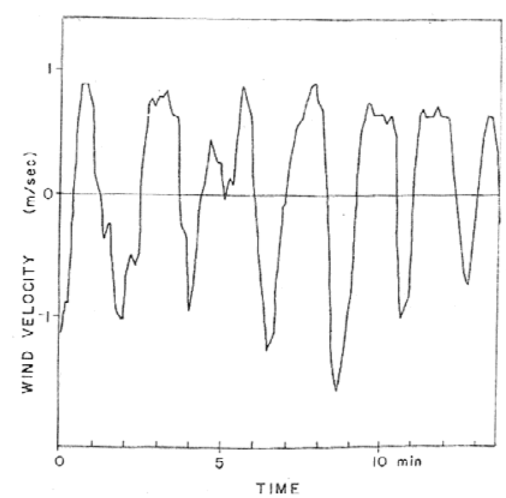

FIG. 3. Portion of a wind-velocity record made within the entrance of Cass Cave.

[8] W. E. Davies, *Caverns of West Virginia* (West Virginia Geological Survey, Morgantown, 1965), pp. 241–243, 264–265.

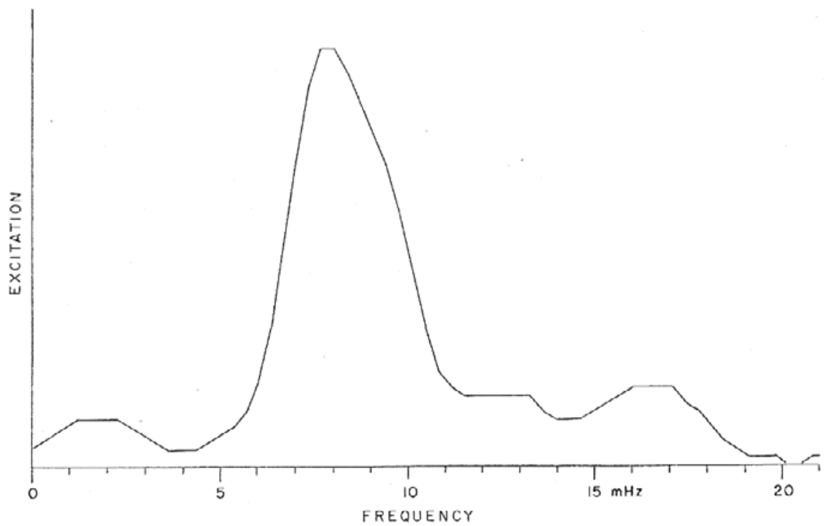

FIG. 4. Spectrum obtained by transforming the velocity record of Fig. 3. The peak lies near the calculated response frequency of the resonator in Fig. 1.

II. EXPERIMENT WITH A DOUBLE RESONATOR

Sinnett Cave, south of Franklin, West Virginia, has also been described by Davies.[8] Like Cass Cave, it consists primarily of a large room, approximately 30 000 m^3, connected to the entrance by a long passage of much smaller cross section (Fig. 5). Schmidt had calculated in 1958 that it should resonate with a period of about $\frac{1}{2}$ min if some way could be found to excite it. In 1958, a very small opening was dug open between the opposite side of the large room and a series of constricted passages connecting to another cave with a second entrance at a much higher elevation. Since that time, a strong and fairly steady thermal air current has blown through the caves from one entrance to the other whenever the ambient temperature differs by more than a few degrees from the nearly constant cave temperature. This thermal wind was exploited to excite the Helmholtz resonance of the large room and the passage joining it to the lower entrance.

FIG. 5. Map of Sinnett Cave, West Virginia, adapted from Davies.[8] A small passage joined to the large room at Point A connects with the smaller room described in the text and shown schematically in Fig. 7.

About 5 m inside the lower entrance, the passage is about 2 m wide and 1 m high. A large sheet of polyethylene was anchored by rocks across the width of this passage so that it could be held easily to block the air flow or quickly dropped out of the way. The vane and recording equipment were placed in a level area 3 m farther into the cave. With the passage open, the wind speed near the vane averaged 3 m/sec on the days when measurements were taken. The vane sensitivity was reduced by diminishing the area of its polyethylene bag and inverting the counterweight. Transients were then generated by obstructing the entrance passage, allowing oscillations throughout the cavern to decay for about 60 sec, then opening the passage abruptly. The velocity records showed signs of a damped sinusoidal resonance following each transient. To diminish the accompanying noise, 26 repetitions of the transient were generated, recorded, and averaged.

The averaged velocity-response function was fitted by an exponentially damped sine wave with the help of a digital computer. The residuals showed that a better fit could be obtained by adding a second sine wave of higher frequency and greater damping. Figure 6 shows the quality of least-squares fit obtained for the data with these two components. The residuals still present were Fourier transformed, collectively and also over shorter intervals, but showed no evidence that a better fit could be achieved with additional components.

The expression representing the velocity following each transient is:

$$V = 2.869 \exp(-0.07147T) \sin(0.1671T) + 0.982 \exp(-0.1709T) \sin(0.5434T - 0.836) + 3.056.$$

The component of lesser frequency lies at $\nu = 26.6$ mHz.

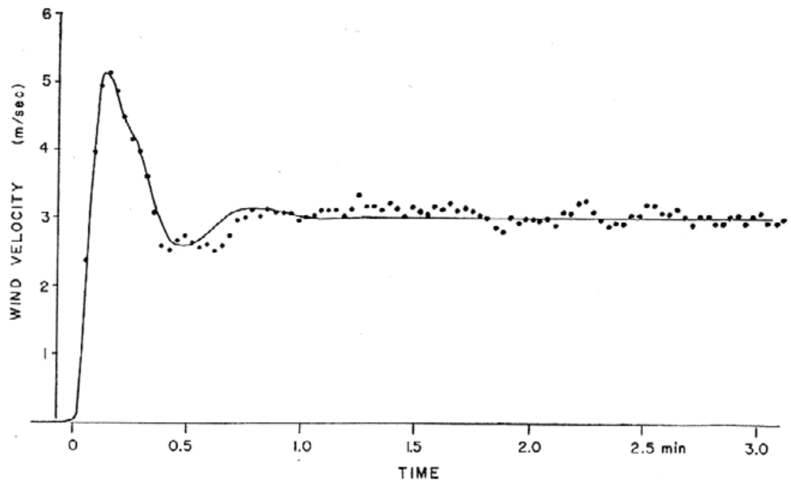

FIG. 6. Average of 26 transient-response records obtained within the entrance of Sinnett Cave (Fig. 5) by blocking the passage and uncovering it abruptly. The smooth curve is the sum of two damped sine waves that provide the closest (least-squares) fit.

An approximate correction for the frequency shift due to damping may be made with the formula:

$$\nu_0^2=\nu^2-(1/2\pi\tau)^2.$$

In this case, $\tau=13.99$ sec, yielding $\nu_0=28.9$ mHz for the equivalent undamped resonance. This result is in good agreement with the value derived by treating the large room and its entrance passage as the volume and neck of a simple resonator.

The second resonant component observed in the transient appears to have smaller amplitude than the first, but the difference is exaggerated by this particular choice of zero for the time scale. A comparison of the two damping factors shows that these observed resonances had more nearly the same instantaneous amplitude when the transient was initiated (at $T=-3.26$ sec). The higher frequency is $\nu=86.5$ mHz, which becomes $\nu_0=90.6$ mHz when damping is removed as before.

The second resonance is also easily related to the structure of this cavern. The passage joining the big room to the higher cave (not shown on map) is not so long as the entrance passage, but its sectional area is an order of magnitude smaller. It opens into a well-defined room of about 1000-m³ volume, or approximately 1/30 the capacity of the large room. A second constricted passage leads from the smaller room to the higher entrance. Acoustically, the portion of the cavern traversed by the thermal air current is, therefore, a double resonator, shown schematically in Fig. 7. Although the presence of the smaller second resonator depresses the resonance of the first, the frequency shift is estimated to be less than 2% because of the great difference in volume between the two rooms. Additional transient records made by interrupting the air flow between the rooms would, of course, isolate the resonant properties of the larger resonator, and the frequency displacement could be measured directly.

One would in fact be able to estimate the volume of the second chamber even if it were not accessible. Although the secular equation that characterizes a coupled oscillator is often quite difficult to solve for the resonant frequencies when damping is large,[9] it may be used rather simply *in reverse* to establish relations among the resonator volumes, effective masses, and

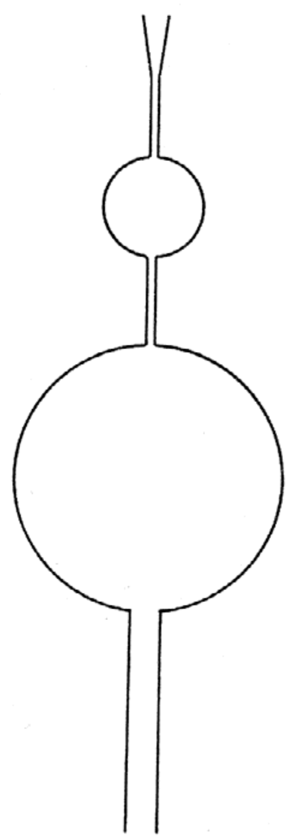

FIG. 7. Schematic geometry of Sinnett Cave, including the smaller room not illustrated in Fig. 5. Transients were produced and measured just inside the lower terminus.

[9] J. C. Slater and N. H. Frank, *Mechanics* (McGraw-Hill Book Co., New York, 1947), pp. 122–142.

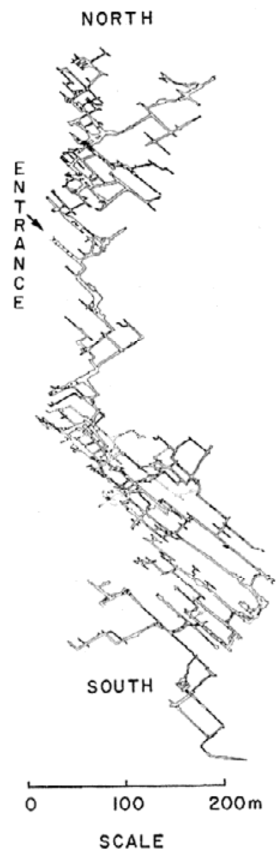

FIG. 8. Map of Breathing Cave, Virginia, adapted from Douglas.[10] Wind velocities were recorded 40 m inside the entrance, at the junction with the south (main) branch of the cave.

damping coefficients when the resonant frequencies and decay times have been measured under the appropriate number of conditions.

III. EXPERIMENT WITH A COMPLEX RESONATOR

Breathing Cave, near Burnsville, Virginia, was the site where repetitive air reversals were first reported by Faust[3]. It has been described by Douglas[10] and by Deike.[11] As shown by the simplified map in Fig. 8, its more than 7 km of passage are interconnected in a complicated pattern. Oscillatory air motions are observed easily at constricted passageways 50 m southeast of the entrance where the northern and southern branches of the cave separate. No other entrances are known. The southern branch equalizes to a change in external barometric pressure with a time constant of a few days. The northern branch responds faster, because its volume is much smaller. For a period of a half day or longer after a barometric step transient, both sections of the cave exhibit wind fluctuations with complicated waveforms. Fluctuations are also observed when irregular winds occur outside the cave.

Some useful wind-velocity measurements were made there by Cournoyer,[5] using suspended vanes of aluminum foil and simple divided scales to measure their deflection. His sampling interval was 15 sec. Fourier transforms of his data, weighted by a triangular apodizing function, suggest that several discrete spectral components can be identified in the range from 2 to 33 mHz. If the same resonant components may be identified on velocity records made at different times, they are probably related to the cave structure and not merely accidental attributes of the local surface winds.

More wind-velocity measurements were made at Breathing Cave for a 2.4-h period in March 1969, with a sampling interval of about 2 sec. The air flow in the connection to the larger (southern) section made many reversals during these observations, and a preliminary Fourier transform of the measurements showed numerous apparent resonances. Since none of the assumed modes of oscillation was continuously excited throughout this entire period, the random changes of phase between successive times of excitation of each mode contributed much spurious spectral structure, particularly in the form of notches subdividing the response peaks. A special reduction technique was devised to overcome the effect of these phase changes.

The power spectrum may be obtained from a large number of velocity measurements by simply computing the Fourier sine and cosine transforms for each component frequency, squaring them, and adding, but other methods are computationally more efficient. Klerer[12] has outlined a useful procedure that can be modified to meet the special requirements of this experiment. First, the autocorrelation of the data is calculated; then the power spectrum is found by a single Fourier cosine transform of the autocorrelation function. This procedure nearly halves the computation time and dispenses with the phase information. For the present purpose, it is convenient to *truncate* the autocorrelation function so that only short-term correlation is considered, the range of time being chosen to be long compared to the periods of the resonances under study, but not much longer than any such resonance might remain continuously excited. The truncated autocorrelation function is weighted by a triangular apodizing function to reduce the side lobes of each spectral line and then converted to a power spectrum by the Fourier cosine transform. The power spectrum obtained in this way shows a spurious high-frequency oscillation, which can be removed rather easily by sampling the spectrum at alternate half-periods of the oscillation and averaging.

The new Breathing Cave data included nearly 4400 velocity measurements. To shorten the computations, only every third point has been used, a total of 1459 values. Each measurement was first diminished by the

[10] H. H. Douglas, *Caves of Virginia* (Virginia Cave Survey, Falls Church, Va., 1964), pp. 129–134.
[11] G. H. Deike III, Master's thesis, Univ. of Missouri, 1960; Nat. Speleol. Soc. Bull. **22**, 30–42 (1960).
[12] M. Klerer, "Computation of Power Spectra," in *Digital Computer Users Handbook*, M. Klerer and G. A. Korn, Eds. (McGraw-Hill Book Co., New York, 1967), Chap. 3.3, pp. 3-53–3-62.

mean velocity to discard the dc level. Then an autocorrelation was calculated over the range of time from zero to about 20 min, was multiplied by the triangular weighting function, and was transformed to obtain a power spectrum from about 0.4 to 90 mHz. The 1459 velocity measurements were subdivided into three equal and nonoverlapping intervals, and a new spectrum was calculated for each interval. All have the same resolving power (roughly 0.6 mHz), since this is now limited by the range of autocorrelation. Figure 9 shows the spectra obtained over the range from 0 to 32 mHz for the three independent sets of data. Although the relative excitation at different frequencies is seen to change considerably from one time interval to the next, there are several frequencies that appear excited in two or more of these spectra. More than 20 have been identified in this range. Additional resonances, particularly at lower frequencies, have been obtained by comparing power spectra computed with a nontruncated autocorrelation (see Table I).

A check was needed to be sure that none of the identified peaks are merely side lobes of true resonances, so a spectrum was calculated from artificial data consisting of a pure sine wave. The spectral response function for the pure sine wave was compared with the cave spectra, and several peaks in each appeared to lie at the same position as side lobes from prominent resonances. These questionable peaks were checked against spectra obtained with other values of the autocorrelation range, for which the side-lobe spacing was different. Only a few of the peaks were spurious. Apparently, the natural oscillations produce smaller side lobes than the artificial sine wave, perhaps because they are naturally apodized by their gradual rise and decay of excitation during the observing period. The number of frequencies listed in Table I is considerably greater than the number of frequency agreements that would be expected by chance if the three spectra were randomly different from each other; most of these frequencies are, therefore, believed to represent true resonances of the cavern. Numerous additional peaks in the spectra may also be true resonances, but more data would be required to test them.

Table I includes additional apparent resonances up to 85 mHz, the effective frequency limit of the 1459 data points used in these calculations. Of the 74 frequencies listed, 34 appear excited in all three of the time intervals separated for calculation; the rest appear in two. Of the 29 frequencies here that fall in the spectral range that can be obtained by transforming Cournoyer's data, 19 are found in his data also. This 65% agreement greatly exceeds the 24% that should agree within this precision, if the frequencies were entirely random. It is especially remarkable since his observation covered only a 17-min interval 14 years earlier.

Breathing Cave must have an enormous number of resonant modes in the higher part of the frequency range considered here, and the unique identification of any one of them with specific structure would be prohibitively difficult. For this reason, the March 1969 measurements have not been transformed to extend these acoustic spectra to their limit of 255 mHz, al-

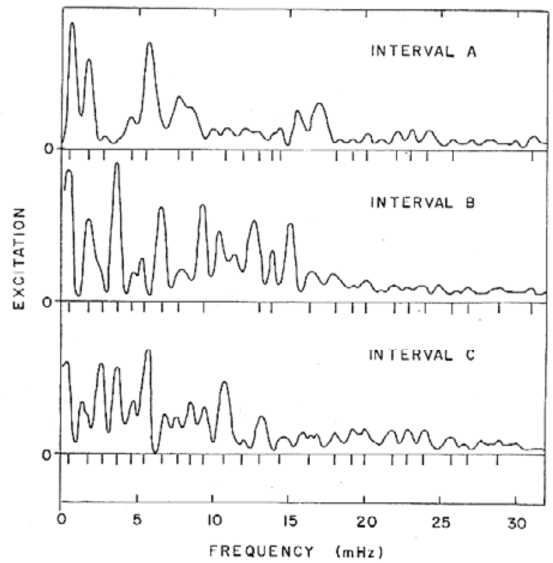

FIG. 9. Spectra from 0 to 32 mHz obtained by transforming three independent sets of velocity records measured at Breathing Cave. Each interval lasted about 50 min, but phase instability has largely been removed by the use of a truncated autocorrelation function with a 20-min range. Short vertical lines mark peaks that can be identified in two or more spectra and probably represent true acoustic resonances of the cavern.

TABLE I. Apparent resonant frequencies (in mHz) in Breathing Cave spectra.

0.4[a]	16.3	34.7[a]	53.1	68.8
1.7[a]	18.0[a]	35.7[a]	55.3	70.9
2.7[a]	19.1[a]	36.6[a]	56.4[a]	71.8
3.7	20.0[a]	38.4	57.3	72.5[a]
4.6[a]	21.9[a]	40.2	58.2[a]	74.3
5.5[a]	22.8[a]	41.2[a]	59.1[a]	75.3
6.6	24.0[a]	42.1[a]	60.1[a]	77.7
7.6[a]	25.7[a]	43.8	60.8[a]	78.1[a]
8.5	26.8	44.6	61.4	78.7
9.3	28.7	44.9	61.8	79.4
11.2	29.8	46.6[a]	64.3	80.8
11.9	31.0[a]	48.0	65.6[a]	83.4[a]
13.1[a]	31.9	49.6[a]	66.4	83.9
13.9	32.8[a]	52.0	67.0[a]	85.1[a]
14.4	33.8	52.6	68.2	

[a] The frequencies identified in all three intervals of observation.

though the data show ample evidence of fluctuations at these higher frequencies.

Since this entire cavern is much shorter than a wavelength at any of the frequencies considered, the method that Rayleigh used for coupled oscillators is appropriate here, but would be prohibitively difficult to apply to this multiply connected geometry. Possibly the passage characteristics throughout the cavern are sufficiently uniform so that the physical parameters could be treated as uniformly distributed for a first approximation. This approach will be attempted when suitable measurements of passage sectional areas have been obtained.

IV. CONCLUSION

These experiments show that low-order acoustic resonances may be detected in large caverns under both natural and artificial means of excitation, in frequency ranges so low that the oscillations are experienced as slowly reversing winds. In some cases, such acoustic observations can provide useful new information about inaccessible cavern structure.