APPENDIX B

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THEORY OF THE CLIMBING KNOT

By William T. Plummer

Abstract

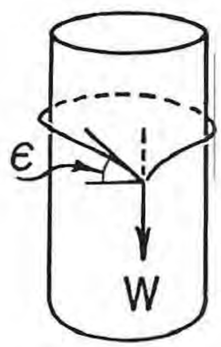
Starting from simplest ideas a mathematical study leads to a quantitative result for the holding power of a climbing knot. Application is made for the case of the prusik knot and for a few other knots in use. A new knot is presented which will hold on any rope, no matter how slippery. Considerations governing the choice of sling material are derived. It is found that a slippery rope is most easily climbed with slippery slings.

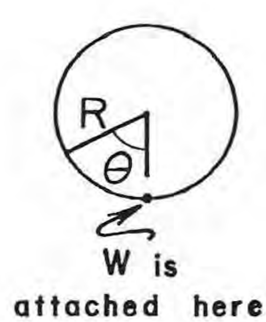
Treatment of a Fixed Loop

The following simplifications are made. The validity of each will be discussed later.

1. The main rope is replaced by a smooth, rigid cylinder.
2. The sling material is completely limp and weightless, but is rigid with respect to a twist about its axis.
3. The sling rope is of much smaller diameter than the rope which is to be climbed.

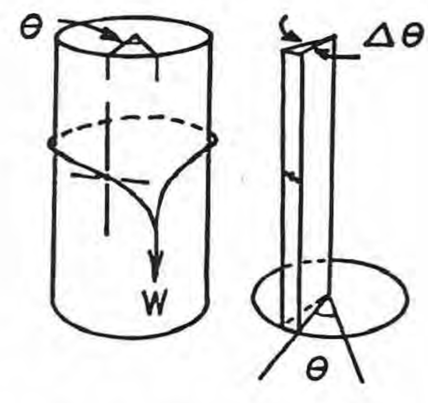
Let us consider the very simplest sort of "knot" which could be used. This is a fixed loop of cord a little larger than the cylinder. When such a loop supports weight W, the point of support will be a cusp in the loop. Both branches will lie at some angle ϵ above the horizontal. The actual curve formed by the loop is more complicated than it looks, but may be found. It will be shown that a loop larger than a definite size will not grip the cylinder, but a loop of any smaller size will. This critical loop size may be characterized by its angle, ϵ.

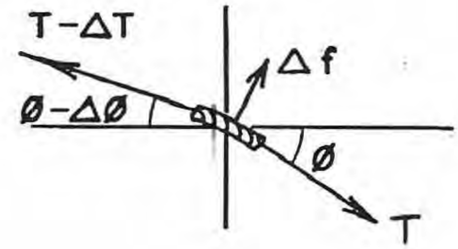




We have to use some kind of coordinate system. Looking down on the cylinder, the angle θ designates a direction out from the center. We may let θ be zero at the point where the weight W is supported, and it will then equal π radians (or 180°) around back at the most distant point. By stating a value for θ, we refer to a particular point on the loop.

One other angle which is important is the tilt of the sling rope, the loop, at each value of θ. We see that this angle, ϕ, is equal to ϵ at $\theta=0$, and decreases to 0 at $\theta=\pi$, around back where the sling loop is horizontal.



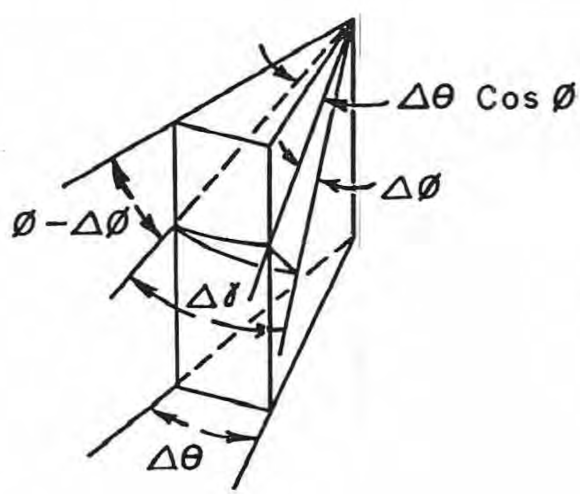


Now look at a very short segment of the loop, at an arbitrary position θ. The little piece extends across a small angle $\Delta\theta$ as shown. We may draw a set of axes right at the segment and show its form. The tensions upon its ends are T and T$-\Delta$T, acting at the angles named. The quantity ΔT is the change in tension from one end of the little piece to the other, and $\Delta\phi$ is the change in tilt from one end to the other. Since the segment is short, both changes may be considered very small.

In the plane tangent to the cylinder at the position θ, there is a force Δf which is perpendicular to the sling and accounts for the curvature. ΔT and Δf are in this plane. To find the values of ΔT and Δf we must approach the problem a little differently.

In general a short element of the loop may be represented as a small arc of a circle. This arc, at any point of the loop, will determine a "plane of curvature." In general, this plane is *not* the same as the plane tangent to the cylinder. In the next few steps we shall find the true plane of curvature at an arbitrary θ.

In the plane of the arc, the angle between the directions of the ends of the element is $\Delta\gamma$. With a little solid geometry, we find that:

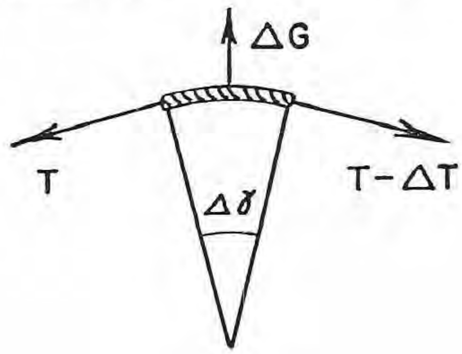


$$(\Delta\gamma)^2 = (\Delta\theta)^2 \cos^2\phi + (\Delta\phi)^2$$

$$\Delta\gamma = \sqrt{(\Delta\theta)^2 \cos^2\phi + (\Delta\phi)^2}$$

or $$\Delta\gamma = \sqrt{\cos^2\phi + \left(\frac{d\phi}{d\theta}\right)^2}\ \Delta\theta$$

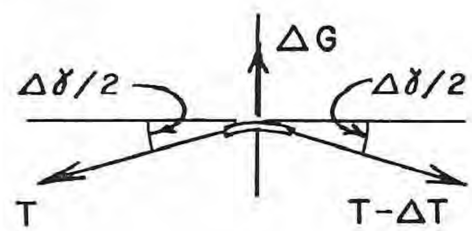
Then in the plane of curvature, we have a situation like this: Here ΔG is the force that is perpendicular to the element of the loop and causes the curvature $\Delta\gamma$.



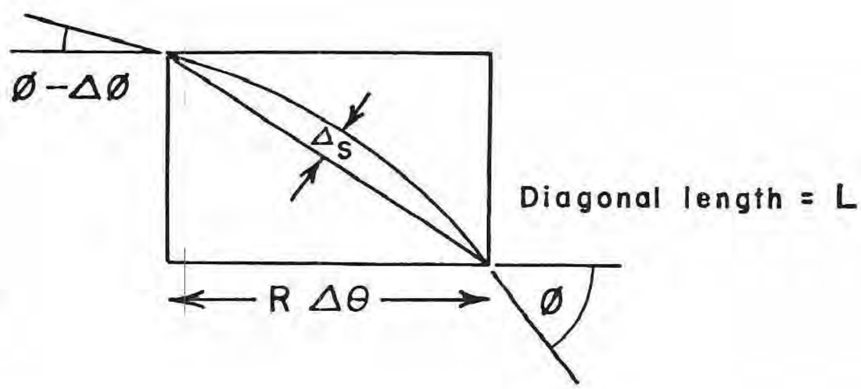
We have seen how $\Delta\gamma$ is related to $\Delta\theta$ and $\Delta\phi$. For the small angle $\Delta\gamma$, we may calculate ΔG easily from the force diagram. Since ΔT is very small,

$$\Delta G \doteq 2T \sin(\Delta\gamma/2) \doteq T(\Delta\gamma)$$

$$\Delta G \doteq T\sqrt{\cos^2\phi + (\frac{d\phi}{d\theta})^2}\ \Delta\theta$$



The force ΔG is perpendicular to the segment of the loop, but may not be perpendicular to the surface of the cylinder. To find its direction, we need more solid geometry. Let's flatten out a little rectangular piece of the cylinder, of width $R\Delta\theta$ and just high enough to contain the little piece of the sling loop.



The diagonal of the rectangle is drawn. The separation Δs between the loop and the diagonal may be found by letting the arc be part of a circle, and noting that $\Delta\phi$ is small.

The radius of a curvature of the arc in this *flattened* area is:

$$\mathcal{R} = \frac{-L}{\Delta\phi} = \frac{-R\,\Delta\theta}{\cos\phi\,\Delta\phi}$$

$$\Delta s = \mathcal{R}(1-\cos\frac{\Delta\phi}{2})$$

$$\cos\frac{\Delta\phi}{2} = 1 - 1/2(\frac{\Delta\phi}{2})^2 + \text{...smaller terms.}$$

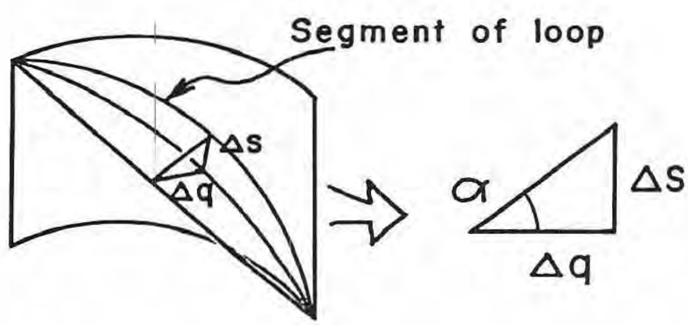
$$1 - \cos\frac{\Delta\phi}{2} \doteq 1/2(\frac{\Delta\phi}{2})^2$$

$$\boxed{\Delta s = (1/2)\,\mathcal{R}(\frac{\Delta\phi}{2})^2 = \frac{-R(\Delta\theta)(\Delta\phi)}{8\cos\phi}}$$

Now put the rectangle back on the cylinder and roll it into its original shape. It will bow outward somewhat, by the amount

$$\Delta q = (1/2)\,R(\frac{\Delta\theta}{z})^2 = \frac{R(\Delta\theta)^2}{8}$$

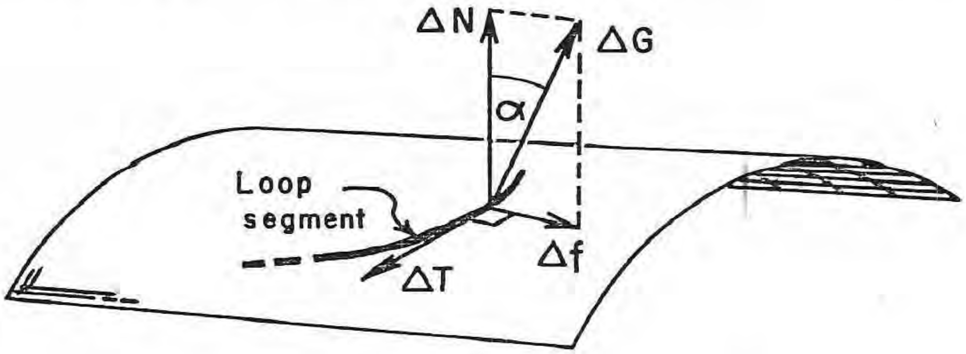
Let us see what the rectangle looks like back in its position:



We get a little right triangle as indicated, when we draw a new straight line connecting the corners of the bent rectangle. This new line lies in the true plane of curvature of the short piece of loop. And more important, the angle α is just the angle between this plane of curvature and the local perpendicular to the cylindrical surface. We see that

$$\tan\alpha = \frac{\Delta s}{\Delta q} = \frac{-R(\Delta\theta)(\Delta\phi)}{8\cos\phi}\cdot\frac{8}{R(\Delta\theta)^2}; \quad \tan\alpha = \frac{-1}{\cos\phi}\left(\frac{d\phi}{d\theta}\right)$$

Now we may resolve ΔG, the force perpendicular to the loop segment in its plane of curvature, into two components, which will be perpendicular and parallel to the cylinder. These components appear as ΔN and Δf, respectively, in the diagram below.



Both Δf and ΔT are in the plane tangent to the cylinder. But ΔN is perpendicular to this plane. The resultant force *in* the plane is the vector sum of Δf and ΔT, and has the magnitude:

$$\sqrt{(\Delta f)^2 + (\Delta T)^2} \leqslant \mu(\Delta N); \; \mu = \text{coefficient of friction}$$

With the proper substitutions this equation will specify the form of the entire sling loop. Since we had

$$\Delta f = \Delta G \sin\alpha$$

$$\Delta N = \Delta G \cos\alpha$$

$$\Delta G = T(\Delta\gamma) = T\sqrt{\cos^2\theta + (\frac{d\phi}{d\theta})^2}\; \Delta\theta$$

The last equation becomes, after substituting for these three quantities and squaring,

$$T^2[\cos^2\phi + (\frac{d\phi}{d\theta})^2](\Delta\theta)^2 \sin^2\alpha + (\Delta T)^2 \leqslant \mu^2\, T^2[\cos^2\phi + (\frac{d\phi}{d\theta})^2](\Delta\theta)^2 \cos^2\alpha$$

We divide through by T^2 and solve for $(\Delta T/T)^2$, and get

$$(\frac{\Delta T}{T})^2 \leqslant [\mu^2 \cos^2\alpha - \sin^2\alpha][\cos^2\phi + (\frac{d\phi}{d\theta})^2](\Delta\theta)^2$$

$$\boxed{0 \geqslant \frac{\Delta T}{T} \geqslant -\sqrt{(\mu^2+1)\cos^2\alpha - 1}\;\sqrt{\cos^2\phi + (\frac{d\phi}{d\theta})^2}\;(\Delta\theta)}$$

This differential equation cannot yet be integrated to give T as a function of θ, because ϕ is an unknown function of θ. In fact, ϕ may not be uniquely determined by θ, so it may be possible for the loop to have a range of shapes, and still not slip. We are interested in only the worst case, however; the point at which the loop just fails to hold. For small values of μ, corresponding to a slippery cylinder, the first radical becomes imaginary. The second radical is always real. Since it is not physically possible for $\Delta T/T$ to be imaginary, the equation shows us the value of μ for which the loop slips.

To keep $\Delta T/T$ real, so that the loop holds, we must set

$$(\mu^2 + 1)\cos^2\alpha - 1 \geqslant 0$$

We note that $\Delta T/T$ is zero for the value of μ which makes this expression zero. Thus a loop at the point of slipping has the same tension all the way around the cylinder. This is a way of saying that the loop has arranged itself so that all available friction is used to prevent slippage, and none is used to change the tension from place to place.

We may write the same expression as

$$\frac{1}{\cos^2\alpha} \leqslant \mu^2 + 1$$

From geometry

$$\frac{1}{\cos^2\alpha} \equiv \tan^2\alpha + 1, \text{ for any } \alpha$$

But

$$\tan\alpha = \frac{-1}{\cos\phi}\left(\frac{d\phi}{d\theta}\right)$$

Then

$$\frac{1}{\cos^2\phi}\left(\frac{d\phi}{d\theta}\right)^2 \leqslant \mu^2, \text{ or } \boxed{\mu \geqslant \frac{1}{\cos\phi}\left(\frac{d\phi}{d\theta}\right) \geqslant -\mu}$$

Right at the point of slipping there is a unique case:

$$\frac{1}{\cos\phi}\left(\frac{d\phi}{d\theta}\right) = \sec\phi\left(\frac{d\phi}{d\theta}\right) = -\mu$$

or

$$\boxed{\sec\phi \, d\phi = -\mu \, d\theta}$$

This is the differential equation for the shape of the loop just at the point of slipping. It may be integrated immediately.

The result is

$$\log \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) + C = -\mu\theta$$

To determine the unknown constant, C, we use the fact that the sling is horizontal (ϕ=0) around back where $\theta=\pi$.

Then

$$\log \tan (\frac{\pi}{4} + 0) + C = -\mu \pi$$

But

$$\tan \frac{\pi}{4} = 1, \text{ and } \log 1 = 0$$

Therefore

$$C = -\mu \pi$$

The equation for ϕ becomes

$$\log \tan (\frac{\pi}{4} + \frac{\phi}{2}) = \mu(\pi - \theta)$$

Or we may write

$$\boxed{\tan \quad (\frac{\pi}{4} + \frac{\phi}{2}) = e^{\mu(\pi-\theta)}}$$

Or equivalently

$$\sin \phi = \tanh \ [\mu(\pi - \theta)]$$

With this expression for ϕ, it is straightforward to obtain the length of the loop by one more integration. The result is

$$\mathcal{L} = 2R \frac{\sinh \mu\pi}{\mu}$$

We will merely note that as μ becomes small, the loop length must approach the cylinder's circumference as a limit, in order to hold.

For our main purpose, we will apply the shape equation to the special point at θ=0, to find the limit $\phi=\epsilon$. Since ϵ is a maximum at the point of slipping, a loop with any value smaller than this will hold. Thus we get

$$\boxed{\tan (\frac{\pi}{4} + \frac{\epsilon}{2}) \leqslant e^{\mu\pi}}$$

$$\textit{or } \sin \epsilon \leqslant \tanh \mu\pi$$

The transcendental equation just written gives either

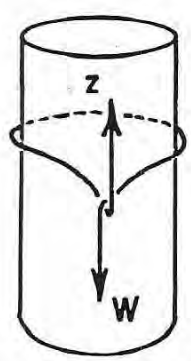
1. The maximum allowable angle ϵ for a given coefficient of friction μ.
2. The minimum allowable friction μ for a specified junction angle ϵ.

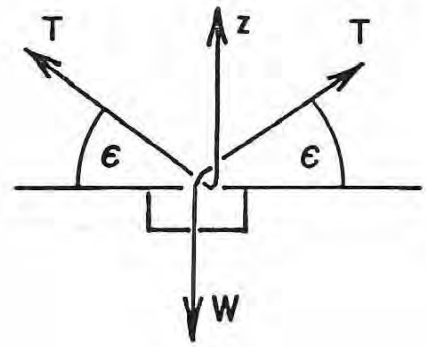
If the fixed loop were of smaller size, or if μ were larger, then ϕ would not be a unique function of θ. Thus we might deform the loop to some slightly different shape, and it would still hold. But we are interested in the limit at which the knot just *fails* to hold. The importance of this case will become apparent as we make use of the results found so far.

To do more, we must introduce some more facts about the physical situation. Instead of a fixed loop, we will consider a single half-hitch on the cylinder. We will find the equilibrium value of ϵ for the half-hitch, and will compare it with the value found above to determine when the half-hitch is stable.

Treatment of a Single Half-Hitch

To keep a single half-hitch from pulling open, we must apply the tension z at the top, in a manner not yet specified. It is not hard to show that z is less than W, and to find the value which ϵ will assume for this case. It will be noted that the knot will change its shape until a stable value of ϵ is found, and then the previous discussion will indicate whether the hitch will slip on the cylinder.





In equilibrium at the junction, the horizontal components of the tensions are equal and opposite. The vertical components will balance, too, when

$$z + 2T \sin \epsilon = W$$

So that the lower sling rope will not slip through the junction, we must have

$$T = We^{-\frac{\mu_s}{\sqrt{2}}\left(\frac{\pi}{2} - \epsilon\right)}$$

(This is just the formula for snubbing around a cylinder. The factor $1/\sqrt{2}$ appears because the lighter rope crosses itself at an angle of 45° at the bend, regardless of ϵ, and $\cos 45° = 1/\sqrt{2}$.) The new coefficient of friction μ_s is for sling rope against *itself*, rather than against the cylinder, and μ_s need not equal μ.

There is a third condition for equilibrium. So that the upper sling rope will not slip through the junction, we use the snubbing formula again and find

$$z = Te^{\frac{-\mu_s}{\sqrt{2}}\left(\frac{\pi}{2} + \epsilon\right)}$$

When we put these last three equations together, we get

$$Te^{\frac{-\mu_s}{\sqrt{2}}\left(\frac{\pi}{2} + \epsilon\right)} = z = W[1-2(\sin\epsilon)e^{\frac{-\mu_s}{\sqrt{2}}\left(\frac{\pi}{2} - \epsilon\right)}]$$

We see that

$$\frac{z}{W} = [\,1 - 2(\sin\epsilon)e^{\frac{-\mu_s}{\sqrt{2}}\left(\frac{\pi}{2} - \epsilon\right)}\,] = \frac{1}{r}$$

For all permissible pairs of ϵ and μ_s, the fraction $1/r$ is less than one. Thus the tension W below the knot has been reduced by the ratio r above the knot, to z. We'll come back to this.

Using the rest of the equation above, and replacing T by its value in terms of W, we get

$$We^{\frac{-\mu_2}{\sqrt{2}}\left(\frac{\pi}{2} - \epsilon\right)}e^{\frac{-\mu_s}{\sqrt{2}}\left(\frac{\pi}{2} + \epsilon\right)} = w\,[1 - 2(\sin\epsilon)\,e^{\frac{-\mu_s}{\sqrt{2}}\left(\frac{\pi}{2} - \epsilon\right)}]$$

Divide out the W's and simplify.

$$e^{\frac{-\mu_s\pi}{\sqrt{2}}} = [1 - 2(\sin\epsilon)e^{\frac{-\mu_s}{\sqrt{2}}\left(\frac{\pi}{2} - \epsilon\right)}]$$

Solve for

$$(\sin\epsilon)\,e^{\frac{-\mu_s\epsilon}{\sqrt{2}}} = \frac{1 - e^{\frac{-\mu_s\pi}{\sqrt{2}}}}{2e^{\frac{-\mu_s\pi}{\sqrt{2}}}} = \frac{1}{2}[e^{\frac{+\mu_s\pi}{2\sqrt{2}}} - e^{\frac{-\mu_s\pi}{2\sqrt{2}}}]$$

Or we could write

$$\boxed{(\sin\epsilon)\,e^{\frac{-\mu_s\epsilon}{\sqrt{2}}} = \sinh\left(\frac{\mu_s\pi}{2\sqrt{2}}\right)}$$

This last equation specifies the single value of ϵ which is permitted for any chosen sling friction μ_s. The sling will slip on itself until this angle develops.

Application of Equations to the Simplest Knot

We have found two transcendental equations for the half-hitch:

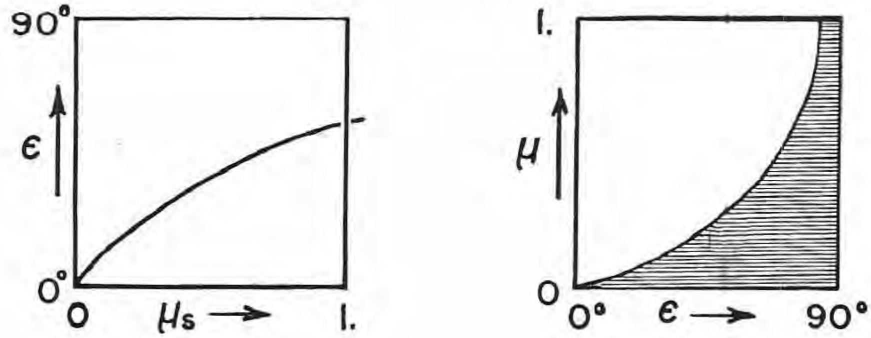
$$(1)\quad \tan\left(\frac{\pi}{4} + \frac{\epsilon}{2}\right) \leqslant e^{\mu\pi}$$

$$(2)\quad (\sin\epsilon)e^{\frac{\mu_s\epsilon}{\sqrt{2}}} = \sinh\left(\frac{\mu_s\pi}{2\sqrt{2}}\right)$$

The first tells the lowest μ we may have with a given ϵ, and the second tells us what ϵ will actually be for a given sling friction μ_S. So starting with knowledge of μ_S for the sling material, we could find ϵ by one equation, and then learn the lowest possible μ from the other equation. That would be the slippery limit for gripping a cylinder.

Transcendental equations cannot be solved in ordinary ways. Usually it is easiest to solve them graphically.

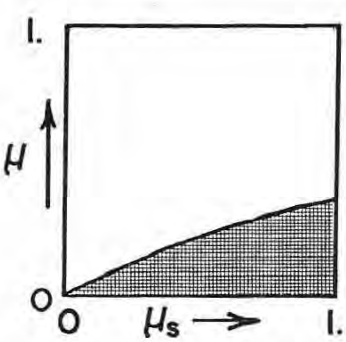
Putting the second equation first, we get the following pair of graphs for the solutions. (These were obtained with the help of preliminary graphs.)



The shaded region indicates prohibited values. Values above it are allowed.

To use these graphs you pick a sling friction μ_S, read the first graph to find the angle ϵ at which the knot will form, and then find this ϵ on the second graph to find the range of permitted values of μ.

In fact, the graphs may be combined so that this information is found in one step. The result is



The curve is remarkably close to a straight line in this region! Now under the condition that a sling alone is characterized by μ_S, and that μ is a joint property of your sling and the rope you want to climb, the condition we have found may be stated simply as:

μ_S must be *less* than 3μ

So to climb slippery rope, it will actually be to your advantage to use slippery slings! This is the reason why new slings are easier to use than old, rougher ones.

Discussion

At first glance it appears rather easy to be sure that μ_S is less than 3μ, just by using slings of the same material as the rope you are climbing. But it is not that simple because μ_S is the coefficient of *static* friction between two parts of the sling, and μ is the coefficient of *moving* friction between the sling and the rope you are climbing. After all, you want to *stop* after your knot has slipped a little. Since static friction may easily be two or three times as great as sliding friction, the condition derived may not be easy to meet.

But before we all go back to climbing ladders we should ask *why* certain values of μ are prohibited, and what happens if we should try to use one.

If your slings are too rough for a particular climb, the knots won't be self-adjusting. This is the difficulty. You could still make the climb, perhaps, by tightening a knot with your fingers each time you let go of it, but the knot wouldn't automatically constrict when you put weight on it. Beyond a second critical value of μ, however, your climbing knot will not stay sufficiently snug even after being tightened by hand, and slippage will take place.

With dry ropes it is almost impossible to fail to meet the condition on μ_S and μ. Observed values of μ_S range from 0.28 for new nylon to 0.89 for well-used manila. Values of μ, measured while moving, range from 0.26 for new manila on new nylon to about 0.65 for old manila on old manila. All of these combinations meet the condition, so observed difficulties in using dry climbing knots on dry ropes are caused entirely by knot stiffness rather than any failure to satisfy the friction requirements.

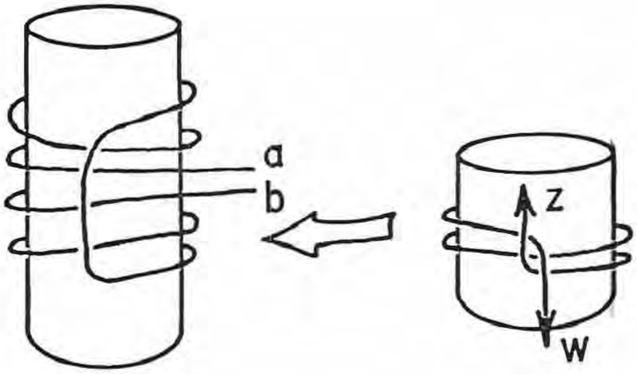
The condition we have derived can easily be made to fail on a main rope coated with mud. The problem is not at all simple, because mud comes in all concentrations, and very dilute mud may actually increase the friction, μ, and make the climbing knot hold tighter. What generally happens, however, is that the film of mud on the main rope develops into a polished surface, with very low μ. If the mud has affected the sling rope to a lesser extent, or in a different manner because of a different composition, then μ may become less than $\mu_S/3$. Now measurements show that μ_S for a manila sling rope will decrease somewhat as the rope is broken in, and then increase. Its lowest value will be about 0.3 for a fresh sling which has been used only enough to lose its stiffness, and the highest value will be about 0.9 for a veteran sling with rough surfaces and plenty of caked dirt. This range indicates that over the course of its lifetime, the absolute holding power of a manila sling will decrease by a factor of three, and near the end it will be capable of holding only clean ropes without help.

From the figures given, nylon would appear to be a good sling material because it is slippery on the surface. But there is a major disadvantage to it. When tension is applied the nylon stretches, and the knot tightens. When tension is removed the nylon shortens and swells. The swelling of the nylon makes most climbing knots tend to jam. In addition, nylon is a little too limp and is more trouble to slide upward. There is need for a strong, slippery, but non-elastic rope material for slings. If your patience is adequate, you may find nylon climbing knots useful for climbing a very muddy rope, when manila knots won't hold. Otherwise, they are a nuisance.

Application to Higher Knots

Now that we have seen how internal friction can affect the holding power of a knot, it shouldn't be surprising that most of the popular climbing knots represent the various ways of getting around this difficulty. Each can be understood a little better in light of the calculation for the half-hitch.

The most familiar and most dependable climbing knot is the prusik knot. The lower half of it is much like the half-hitch we have been discussing, except that there is an extra turn.

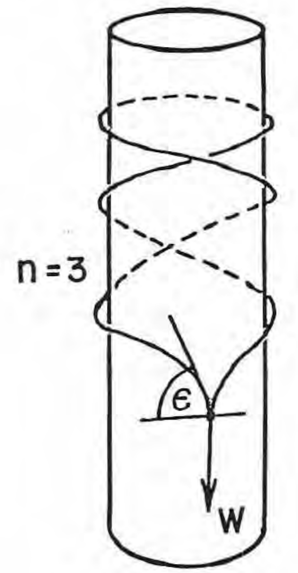


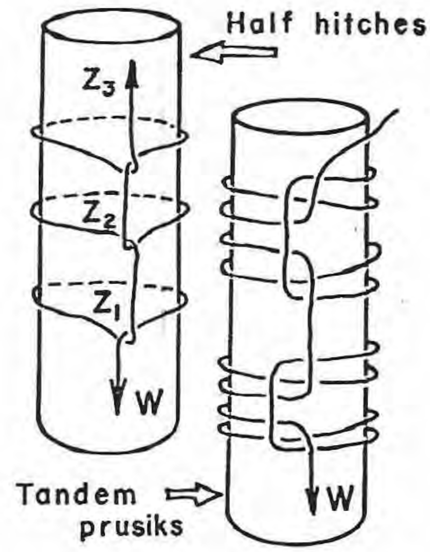
As before, z is less than W by a ratio controlled exponentially by μ. With the two loops the ratio is approximately $R=r^2$, since the tension at the top of the first loop is diminished again by the same ratio. (A third loop would not, however, be much improvement.) We find that the ratio R is increased by some other features as well. Where the vertical parts meet, there is some pinching of the inner turn against the main rope. The part of the knot connecting upper and lower halves serves two purposes. First, it provides the tension z, arising from the drag of the upper two turns on the main rope. Second, it pulls all four turns together to make ϵ *smaller than the value it would have with a half-hitch in equilibrium.* When this happens, the condition on μ becomes less demanding, and it is possible to climb on a more slippery rope. (see graph of μ against ϵ.)

If you have trouble making a prusik knot grip, it will help to adjust things to increase the tension in the lower part of the knot (b) and decrease the tension in the upper part (a). In fact, the upper end (a) could just as well be entirely loose.

Another climbing knot sometimes used is formed by wrapping the sling around the main rope in a criss-crossing fashion. In this case we refer back to the treatment of the fixed loop. Angle ϵ is determined as soon as this climbing knot is tied, and then the minimum permissible value of μ may be obtained from the first of the differential equations. In this case ϕ will be zero not for $\theta=\pi$, but for θ equal to some odd multiple of π. In the picture, ϕ is zero when $\theta=3$. Thus μ cannot be found directly from the graph. But we see from the equation that μ is effectively multiplied by n when the sling encircles the main rope (or the cylinder) n times, as compared with the single fixed loop first studied. There are then two important advantages: a rather small value of ϵ may be used in each turn, and μ is multiplied by the number of turns. Since the holding power is exponential in μ, the holding power of a knot with n turns will go roughly as r^n. I say "roughly" because the sling will tend to stick to itself at the points of crossing, and will not readily conform to the most efficient shape. This knot can lose its form more easily than a prusik knot would, and it may slip unexpectedly.

Other knots have been constructed on this general design, some having the sling rope doubled as it winds around the main rope. Because the knot is loose and open, it may be tied with some types of nylon sling material without extreme jamming.





A while ago the ratio

$$\frac{z}{W} = 1 - 2(\sin\epsilon)\, e^{\frac{-\mu_s}{\sqrt{2}}\left(\frac{\pi}{2} - \epsilon\right)} = \frac{1}{r}$$

was derived for the half-hitch. Now in the unhappy limit when μ_s and μ are very small, ϵ also will be small. We may then say approximately,

$$\epsilon \doteq \mu\pi, \quad \sin\epsilon \doteq \epsilon \doteq \mu\pi$$

$$\mu_s \doteq 2\sqrt{2\mu}, \quad e^{\frac{-\mu_s}{\sqrt{2}}\left(\frac{\pi}{2} - \epsilon\right)} \doteq e^{-2\mu\left(\frac{\pi}{2} - \mu\pi\right)} \doteq 1 - \mu\pi$$

$$\frac{1}{r} = 1 - 2\mu\pi(1 - \mu\pi) = 1 - 2\mu\pi + \frac{4\mu^2\pi^2}{2} \doteq e^{-2\pi\mu}$$

$$\boxed{\frac{z}{W} \doteq e^{-2\pi\mu} < 1}$$

We now see that the ratio z/W is less than one for any possible μ. The form of the last expression happens to be identical to the familiar snubbing formula. The tension reduction with the half-hitch, in the limit when μ is made small, is just the reduction which would be found if a length of sling rope were snubbed one full turn (2π) around the main rope (or cylinder).

If we now set up several half-hitches in tandem, then

$$\frac{1}{r} = \frac{z_1}{W} = \frac{z_2}{z_1} = \frac{z_3}{z_2} = \ldots = \frac{z_n}{z_{n-1}}$$

Thus with n half-hitches, the ratio obtained is the product of these, or

$$\frac{z_n}{W} = \left(\frac{1}{r}\right)^n = e^{-2\pi\mu n} = \frac{1}{R}$$

Even with a small value of μ, the product ($n\mu$) may be large enough so that the knot holds well. All we have to do is find a way to provide that final small tension z_n at the top, so that the total knot will keep its shape. This is done just by adding one more half-hitch. The drag it provides will be enough, if the necessary z_n has been made small. To get rid of the upper end, *entwine* it with the loop of the last half-hitch. Do NOT tie it to the lower end of the sling, as a pull on the *top* of the knot can cause it to slip.

As a practical suggestion, make alternate half-hitches mirror images of each other, to cancel out their tendency to twist on the rope. When this is done, the knot will keep its shape well.

Try this knot sometime when you are experimenting with ropes. It may save you the shame of a rescue someday when your main rope is muddy and nothing else will hold. It is less convenient to use than a prusik knot, and takes up more room, but by adding turns, you can make it grip *anything*.

Along this line, two prusik knots properly tied in tandem will hold on nearly anything. The upper end is just left free. (See next page.) In a typical case the drag provided by the upper half of a prusik knot may be only one pound. If the ratio of a half-knot is R=20, then the entire knot might support only 20 pounds. But two knots in tandem would support 1x20x20x20, or 8000 pounds. The rope would break before the knots could slip!

Either of these combination knots, the running half-hitch or the tandem prusik knot, could eliminate most of the vertical cave rescues made necessary by slippery ropes. I've used the tandem prusik knots to climb a 30-foot piece of brand new quarter inch nylon rope, with slings made of 3/8-inch manila. The knots were in no danger of slipping: they were even a little hard to move.

In these cases we gain remarkable gripping power by making use of the fact that the mechanical advantage of climbing knot is multiplicative, and not merely additive.

As with all aspects of vertical rigging, it is a good idea to try out knots like these while you are above ground. Learn what they will do for you, and what their limitations are, before you use them in a cave.

The assumptions made initially are worth a short note. First, we called the main rope a smooth, rigid cylinder. Its lack of rigidity and its structure will be of some help in holding a climbing knot. The depression of the rope under the constriction of the knot will increase μ. Second, the sling material was considered to be ideally limp, weightless, and non-twisting. Its weight is negligible, and each popular knot design is arranged to cancel the effects of twisting. You recall that in the running half-hitch, the alternate loops are made mirror images of each other for this purpose. But it is well known that sling rope is not very limp, until has been "broken in". If the sling is stiff, it will not contact well along the entire angle assumed, and the result will be a decrease in the effective value of μ, and perhaps a loss of grip. The stiffness begins to become important when you bend the sling rope about a radius comparable to its own. To avoid this, use a sling rope a size or two smaller than your main rope, so that the bending radius is proportionately larger. Both 5/16-inch and 3/8-inch slings work well on 1/2-inch rope.

The stiffness consideration does not change the discussion of the half-hitch where it crosses itself, for there the ropes cross at 45°, and the effective radius of bending is twice the radius of the rope. In any case, a little stiffness would be an advantage here.

There isn't space to discuss the method of prusik climbing, and our concern has been entirely with knots which can be used. The method has been developed to great efficiency, and is actually easier than ladder climbing over a long distance. The central idea is to balance properly so that your legs do all the work of climbing and your arms do almost nothing. This efficiency requires slings of exactly the right lengths *for you*, and these may be calculated from the formulas in the June 1962 NSS NEWS, page 70. Other information may be found in back issues of the *Baltimore Grotto News* or the January 1966 NSS BULLETIN.

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